Development of a Simplified Relationship Between Uniaxial Creep, Stress Relaxation, and Constant Strain-Rate Results for Viscoelastic Polymeric Materials

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Received 30 August 2000; accepted 31 October 2000

ABSTRACT: This publication introduces a new mathematical model to describe a definitive relationship between constant strain-rate, creep, and stress-relaxation analysis for viscoelastic polymeric compounds. This new concept is especially significant since it adequately describes all the important characteristics of both creep and stress relaxation in the same model. In particular, all three phases of creep (i.e., primary, secondary, and tertiary) can be described adequately using this model. This new model for polymeric materials also indicates that yielding for constant strain-rate measurements and the inception of tertiary creep appear to be directly related and may, in fact, be manifestations of the same phenomena. The initial buildup of stress followed by the drop off in stress as a function of time for stress relaxation is also adequately described. This new formulation approach also offers a reasonably simple process in which to shift from a constant strain-rate configuration to a creep calculation or stress-relaxation configuration without changing formulation considerations. Most importantly, this model can be used to make a transition from one of these stress-configuration modes to another without stress or strain discontinuities. It is hoped that this analysis approach will open new doors for the design of plastic products for both short-term and long-term applications. © 2001 John Wiley & Sons, Inc. J Appl Polym Sci 82: 527–540, 2001

Key words: stress versus strain; constant strain rate; creep; stress relaxation; viscoelastic/polymeric materials

INTRODUCTION

Many authors have previously addressed models to predict the very practical viscoelastic mechanical properties of polymeric materials that include creep,[1,2] stress-relaxation,[3] and constant strain-rate measurements.[4–7] However, with the advent of the extended use of finite-element analysis with polymeric compounds[8] and composites,[9] a simple analysis approach that relates all three of these separate processes in a simplified mathematical approach has been particularly needed in recent years. This is particularly true since many applications for polymers and polymer formulations are often subjected to different types of stress configurations in the course of a given application analysis. For example, a gasket or a seal may be subjected to a simulated constant strain-rate operation when the gasket or seal is installed, but then it must function in a stress-relaxation mode for the rest of its life. While some limited efforts have attempted to describe two or more of these processing techniques in a unifying formulation,[3,10] most of the effort over the years has been to simulate uniaxial creep,[1,2] stress-relaxation,[3] or constant strain-rate data[4–7] separately. A very simple technique to approximate all

Journal of Applied Polymer Science, Vol. 82, 527–540 (2001)

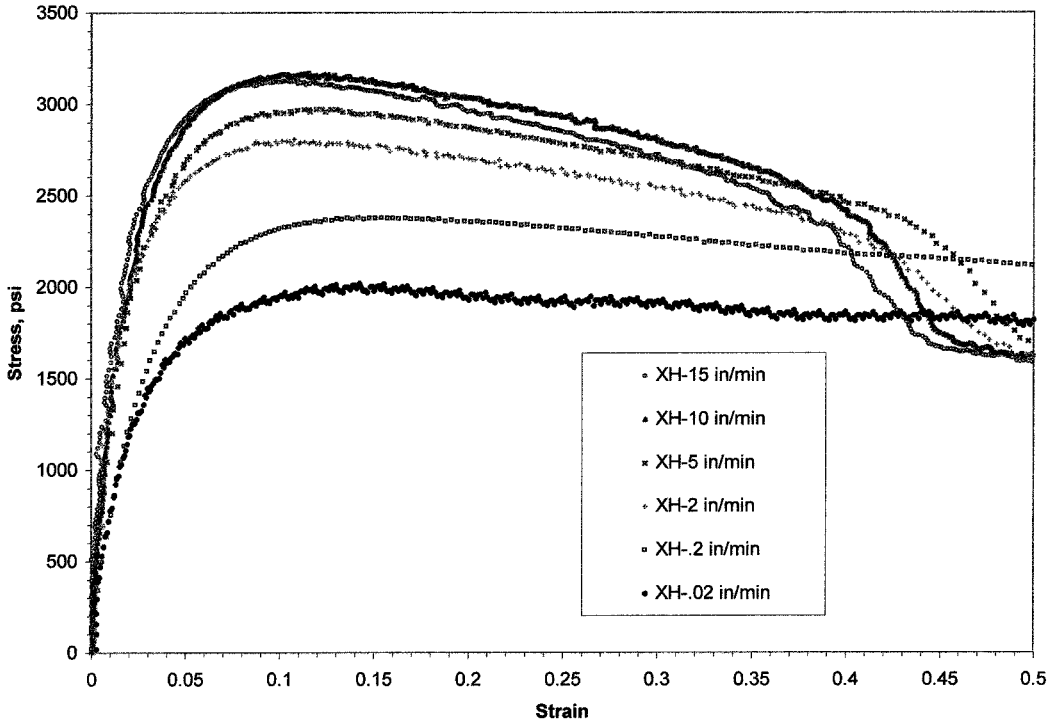

Figure 1 Stress versus strain at different strain rates for polyethylene.

three in one simulated approach is not currently apparent in the literature. This study will address a new analysis approach to represent all three of these measurements on a typical plot of stress versus strain for a series of constant strain-rate measurements. This new evaluation process will then address an estimate of creep, stress-relaxation, or a constant strain-rate analysis as required for a given application.

Stress Versus Strain at Constant Strain Rate

The effect of a series of tensile strain rates on the stress versus strain curves of a typical polyethylene is indicated in Figure 1. These tensile data were generated on an MTS Q-Test universal testing machine using injection-molded polyethylene dog-bone samples prepared and supplied by Westlake Polymers. The polymer used was a Westlake high-density polyethylene (0.954 g/cc) designated as HM4000AA with a melt index of 6.8 (ASTM D1238 condition 190°C/2.16 kg). An extensometer was used to evaluate strain more accurately for these stress–strain curves.

Some specific points of interest that can be gleaned from the results in Figure 1 include

- The yield strength and modulus were both found to increase with an increase in the strain rate.
- The elongation to yield varied only slightly as the strain rate was varied over wide ranges.

These results illustrate several of the concepts indicated in the model to be developed in this study. A more complete development of the polyethylene experimental results indicated in Figure 1 will be submitted in a later publication following the theoretical concepts introduced in this study.

The results shown in Figure 2 were generated using the model to be developed in this study. The constants used to plot the results in Figure 2 were found to be characteristic of representative ABS-formulated polymeric materials. Note that the general shape of the curves in Figure 2 are similar to the experimental results indicated in Figure 1. Consequently, the results in Figure 2 will be used to illustrate the theoretical concepts to be developed in this study.

Since stress relaxation is defined as the change in stress as a function of time required to maintain a constant level of strain, then, conceptually, stress relaxation could be developed from the data

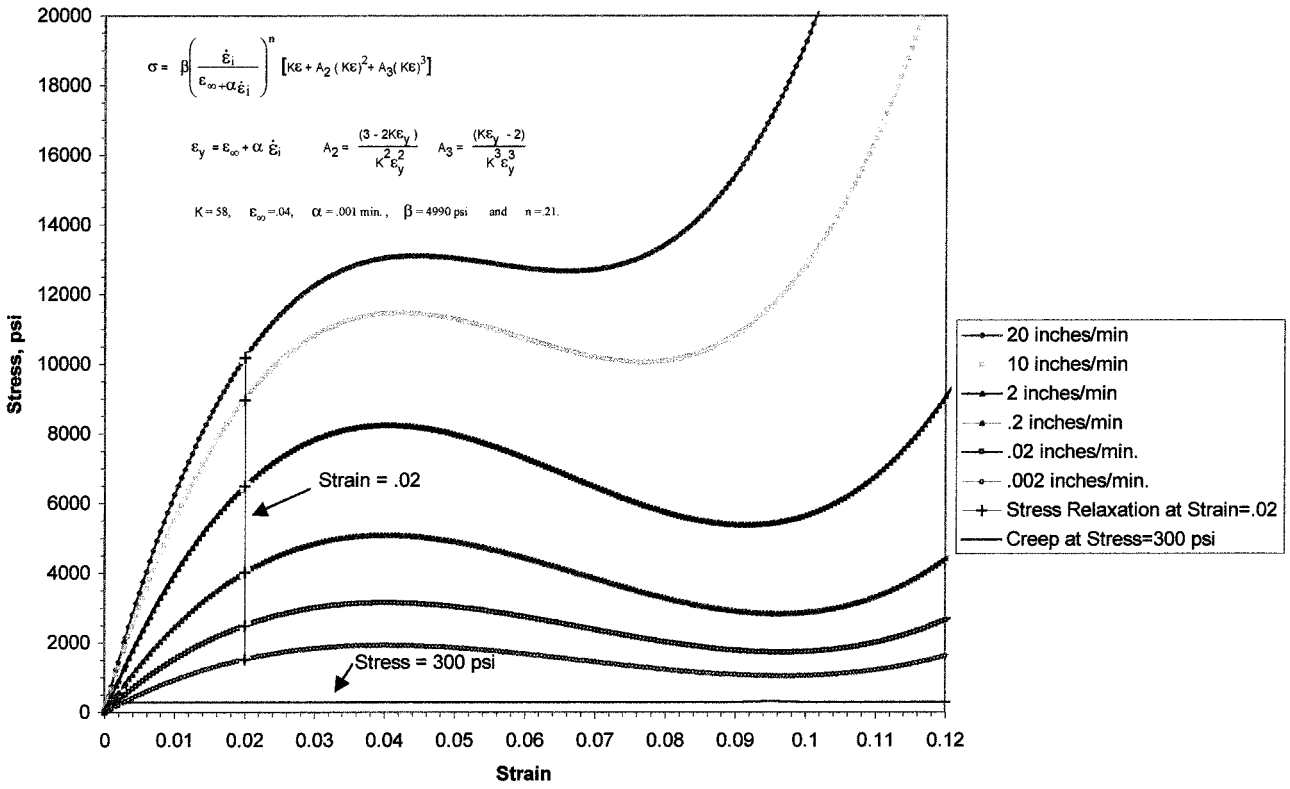

Figure 2 Calculated stress versus strain at various strain rates using model from this study with indications of examples for creep and stress-relaxation contributions.

in Figure 2 by connecting the locus of points on all of the stress-versus-strain curves at the same strain level ($\varepsilon = 0.02$) as indicated. Similarly, since creep can be defined as the change in strain as a function of time at constant stress, then a uniaxial creep curve should also be able to be developed from all the points indicated as a straight line at a constant stress level (i.e., $\sigma = 300$ psi) shown in Figure 2.

Strain-to-Yield Considerations

To more clearly delineate the processes of stress relaxation and creep as indicated in Figure 2, it is initially informative to address the strain at yield, ε_y, for constant strain-rate measurements. Preliminary experimental measurements by this author as well as others in the literature[3,10] have found that the strain at yield, ε_y, generally has been found to be a linear function of the characteristic strain rate, $\dot{\varepsilon}_i$, for constant strain-rate measurements as

$$\varepsilon_y = \varepsilon_\infty + \alpha\dot{\varepsilon}_i \tag{1}$$

where ε_∞ is the limiting strain to yield when the strain-rate approaches an infinitely small value ($\dot{\varepsilon}_i \to 0$) and α is simply a small proportionality constant. Brown[5] was one of the first to describe a relationship of the type indicated by eq. (1) after first proposing the following general relationship between stress and strain for an ideal solid:

$$\frac{\sigma}{\tau_\varepsilon} + \dot{\sigma} = E_u\left(\frac{\varepsilon}{\tau_\sigma} + \dot{\varepsilon}\right) \tag{2}$$

where E_u is the unrelaxed modulus; τ_σ, the characteristic relaxation time for creep at some stress σ; τ_ε, the characteristic relaxation time at some strain ε; σ, the stress; $\dot{\sigma}$, the applied stress rate; ε, the strain; and $\dot{\varepsilon}$, the applied strain rate. Two forms for eq. (1) were obtained by starting from eq. (2) with different assumptions. The first relationship at the yield point was found after making the following assumptions at the yield point:

If $\dot{\varepsilon}$ = constant and $\dot{\sigma} = 0$ at the yield point, then eq. (2) simplifies to

$$\varepsilon_y = \left(\frac{\sigma_y}{E_u}\right)\left(\frac{\tau_\sigma}{\tau_\varepsilon}\right) - \dot{\varepsilon}\tau_\sigma \tag{3}$$

However, if $\dot{\sigma} = \dot{\varepsilon}\sigma$ at the yield point, then eq. (2) simplifies to

$$\varepsilon_y = \left(\frac{\sigma_y}{E_u}\right)\left(\frac{\tau_\sigma}{\tau_\varepsilon}\right) - \dot{\varepsilon}\tau_\sigma\left(1 - \frac{\sigma_y}{E_u}\right) \tag{4}$$

Both eqs. (3) and (4) predict that the constant α would be negative. Brinson and DasGupta[3] pointed out that Crochet[11] predicted, theoretically, that the yield strain should decrease with increase in the strain rate. For the results indicted in Figure 1, it is apparent that α is indeed negative for this polyethylene data. However, Malpass[10] and this author found that for most ABS materials the strain to yield increases as the strain rate increases, which would make α positive. In addition, Brinson and DasGupta[3] also found that, experimentally, the yield strain increased with increase in the strain rate for polycarbonate.

Brown[5] also rearranged both eqs. (3) and (4) to yield the following:

$$\left(\frac{\sigma_y}{E_u}\right) = \varepsilon_y\left(\frac{\tau_\sigma}{\tau_\varepsilon}\right) + \tau_\varepsilon\dot{\varepsilon} \tag{3A}$$

$$\left(\frac{\sigma_y}{E_u}\right) = \left(\frac{\varepsilon_y\left(\frac{\tau_\sigma}{\tau_\varepsilon}\right) + \tau_\varepsilon\dot{\varepsilon}}{1 + \tau_\varepsilon\dot{\varepsilon}}\right) \tag{4A}$$

Note that for both eqs. (3A) and (4A) as $\dot{\varepsilon} \to 0$ the term σ_y/E_u approaches a constant value. Brown[5,12,13] indicated in several articles that within a given class of polymers, and particularly for the same polymer formulation, that the ratio of σ_y/E_u is approximately a constant. In addition, both Buchdahl[6] and Robertson[14] also indicated that the ratio of the yield strength to the elastic modulus is approximately a constant for similar polymer formulations.

Development of a New Model Relating Constant Strain-rate, Creep, and Stress-relaxation Measurements

Based on the considerations already presented, it is proposed that the most general equation to fit a stress–strain curve can be written as

$$\frac{\sigma}{\sigma_y} = \frac{A_1 E\varepsilon}{\sigma_y} + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 + A_4(K\varepsilon)^4 + A_n(K\varepsilon)^n \tag{5}$$

or

$$\frac{\sigma}{\sigma_y} = K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 + A_4(K\varepsilon)^4 + A_n(K\varepsilon)^n \tag{5A}$$

where $K = A_1E/\sigma_y \approx$ constant for a series of strain rates for the same polymer formulation and A_2, A_3, . . . A_i = variable constants for a series of strain rates for the same polymer formulation.

As $\varepsilon \to 0$, then the limiting slope, σ_yK, of eq. (5) approaches the modulus, E, when

$$\sigma_yK \to A_1E \to E \qquad \text{as } A_1 \to 1 \tag{6}$$

The strain, ε, in eq. (5) should be positive in a tension mode and negative for the compression mode. The analysis in this study assumed the strain, ε, to be positive and in a tension mode. The required modifications for the compression condition will be left to the reader.

At this point, it can easily be shown that the exponential

$$\frac{\sigma}{\sigma_y} = 1 - \exp(-K\varepsilon) \tag{7}$$

is one of the simplest equations consistent with the infinite series described by eq. (5). However, one major disadvantage of eq. (7) is that it does not have a well-defined or a finite yield point. Therefore, the example to be discussed in some detail in this article will use only the first three constants in eq. (5) as

$$\frac{\sigma}{\sigma_y} = K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 \tag{8}$$

If it is assumed that $A_1 = 1.0$, then $K = E/\sigma_y$, which, according to Brown[5,12,13] and several other authors,[6,14] is normally a constant for a given polymer formulation that typically ranges from 40 to 60. Again, the ratio of the modulus to the yield stress for the polyethylene indicated in Figure 1 generally had an average value of $K = 45.2$, which is very consistent with the results found by others. Some recent measurements[15,16] of the modulus and yield strength for several polymeric materials are shown in Table I. In general, the average value ratio of the modulus to tensile strength for these data does appear to be reasonably consistent with the general guidelines indi-

Table I Measurements of Modulus and Yield Strength for Several Polymeric Materials

Polymer	Modulus, E	Yield Strength, σy		
Rubber-modified OSA Polymers	(Mpa)	(Mpa)	Ratio K $K = E/\sigma y$	Reference
Sample A	1500	35	42.9	15
Sample B	1400	30	46.7	15
Sample C	1200	23.7	50.6	15
Sample D	1450	29	50.0	15
Sample E	1600	32.5	49.2	15
Sample F	2400	41.5	57.8	15
Sample G	2350	43.5	54.0	15
Sample H	1450	30	48.3	15
Sample I	1500	30.6	49.0	15
Sample J	1000	21.6	46.3	15
Sample K	1800	35	51.4	15
Sample L	1400	28.6	49.0	15
Sample M	1850	34	54.4	15
Sample N	2250	43.5	51.7	15
Sample O	1600	26.5	60.4	15
Sample P	1900	26.7	71.2	15
Sample Q	1000	16.7	59.9	15
Average			52.5	
High-temperature Thermoplastics	(ksi)	(ksi)		
Polyimide	580	16	36.3	16
Polyimide	540	17.3	31.2	16
Polyimide	546	14.8	36.9	16
Polyetherimide	430	15.2	28.3	16
Polyamideimide	400–667	9.2–13.0	43.5–51.3	16
Polyarylimide	460	15	30.7	16
Polyimidesufone	719	9.1	79.0	16
Polysulfone	360	10.2	35.3	16
Polyarylsulfone	310	10.4	29.8	16
Poly(arylene sulfide)	470	14.5	32.4	16
Poly(phenylene sulfide)	630	12	52.5	16
Poly(ether sulfone)	380	12.2	31.1	16
Poly(ether ketone)	580	16	36.3	16
Poly(ether ether ketone)	450	14.5	31.0	16
Poly(arylene ketone)	360	12.7	28.3	16
Liquid crystal polymer	2400	20	120.0	16
Average			42.9	

cated by Brown. The two other conditions required to evaluate the constants A_2 and A_3 in eq. (8) would include the following:

By definition: $\sigma = \sigma_y$ when $\varepsilon = \varepsilon_y$.

Second condition: $d\sigma/d\varepsilon = 0$ at $\sigma = \sigma_y$ when $\varepsilon = \varepsilon_y$.

Using these conditions, it can be shown that if $K\varepsilon_y \leq 3$

$$A_2 = \frac{(3 - 2K\varepsilon_y)}{K^2\varepsilon_y^2} \tag{9}$$

$$A_3 = \frac{(K\varepsilon_y - 2)}{K^3\varepsilon_y^3} \tag{10}$$

Thus, if $[d(\sigma/\sigma_y)]/(d\varepsilon) = 0$ and if $K\varepsilon_y \leq 3$, then the two extrema at $\varepsilon = \varepsilon_1$ and $\varepsilon = \varepsilon_2$ can be found to yield a maximum at

$$\sigma_1 = \sigma_y \text{ at } \varepsilon_1 = \varepsilon_y \tag{11}$$

and a minimum at

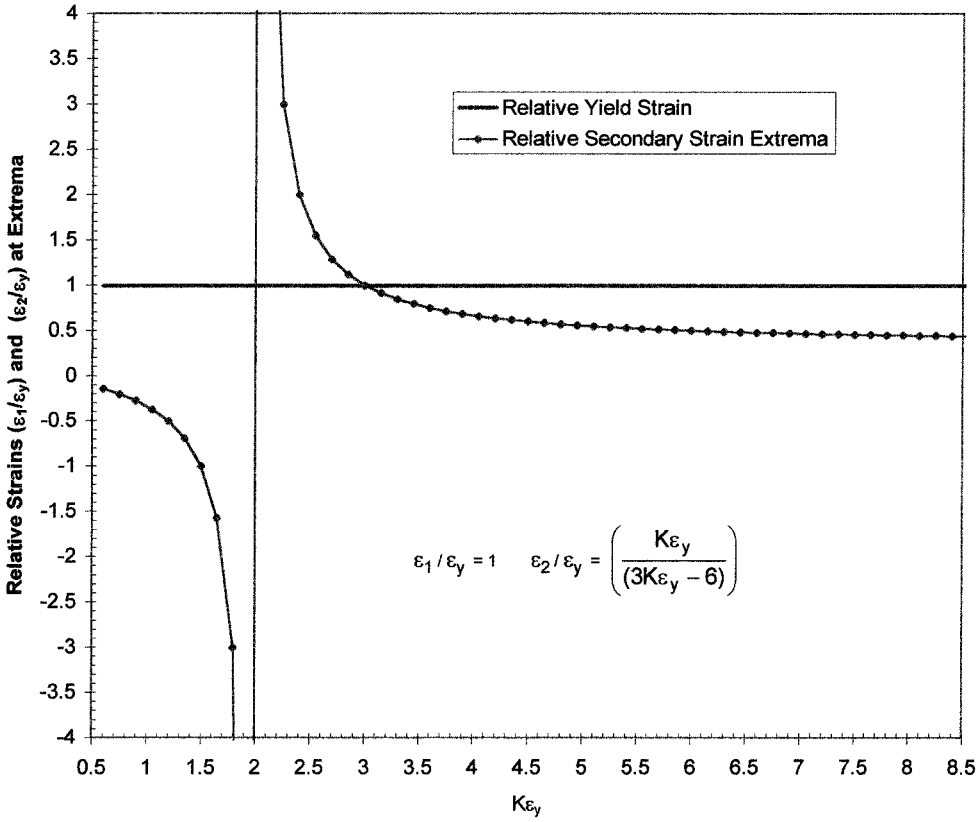

Figure 3 Relative yield strains ($\varepsilon_1/\varepsilon_y$) and relative secondary strain extrema ($\varepsilon_2/\varepsilon_y$) versus $K\varepsilon_y$.

$$\sigma_2 = \sigma_y\left(\frac{K^2\varepsilon_y^2(4K\varepsilon_y - 9)}{27(K\varepsilon_y - 2)^2}\right) \text{ at } \varepsilon_2 = \varepsilon_y\left(\frac{K\varepsilon_y}{(3K\varepsilon_y - 6)}\right) \tag{12}$$

Note that $\varepsilon_1 = \varepsilon_2 = \varepsilon_y$ when $K\varepsilon_y = 3$. This is clearly shown in Figure 3, where ($\varepsilon_2/\varepsilon_y$) is plotted as a function of $K\varepsilon_y$. Similarly, note that $\sigma_1 = \sigma_2 = \sigma_y$ when $K\varepsilon_y = 3$ as indicated in Figure 4, where (σ_2/σ_y) is plotted as a function of $K\varepsilon_y$. It is also clear that $\sigma_2 \leq \sigma_1 = \sigma_y$ as long as $K\varepsilon_y \leq 3$. It is also very clear that if $K\varepsilon_y > 3$ additional changes need to be made to the formulation included in the main body of this article. For completeness, the primary formulation modifications to achieve the primary objectives indicated in this study when $K\varepsilon_y > 3$ are summarized in the Appendix. Using the principles discussed here, the special case summarized in the Appendix can be modified to address the constant strain rate, creep, and stress relaxation with little disruption in content.

Another relationship between stress and time can also be introduced by noting that the stress relaxation of the yield point can be addressed using the following simple relationship currently included in ASTM D2837 –98a (Standard Test Method for Obtaining Hydrostatic Design Basis for Thermoplastic Pipe Materials):

$$\sigma_y = \frac{\beta}{t_y^n} \tag{13}$$

where σ_y is the yield point; t_y, the time to yield; and β and n, constants. This relationship was also used by Reinhart[17] to predict long-term failure stress (which is normally close to the stress evaluated from the stress relaxation of the yield stress) as a function of time.

The calculated values of strain, ε, shown in Figure 2, can also be evaluated on a time scale as indicated in Figure 5 by noting that the time, t, to reach a given strain, ε, can be evaluated from the characteristic strain rate, $\dot{\varepsilon}_i$, as

$$t = \frac{\varepsilon}{\dot{\varepsilon}_i} \tag{14}$$

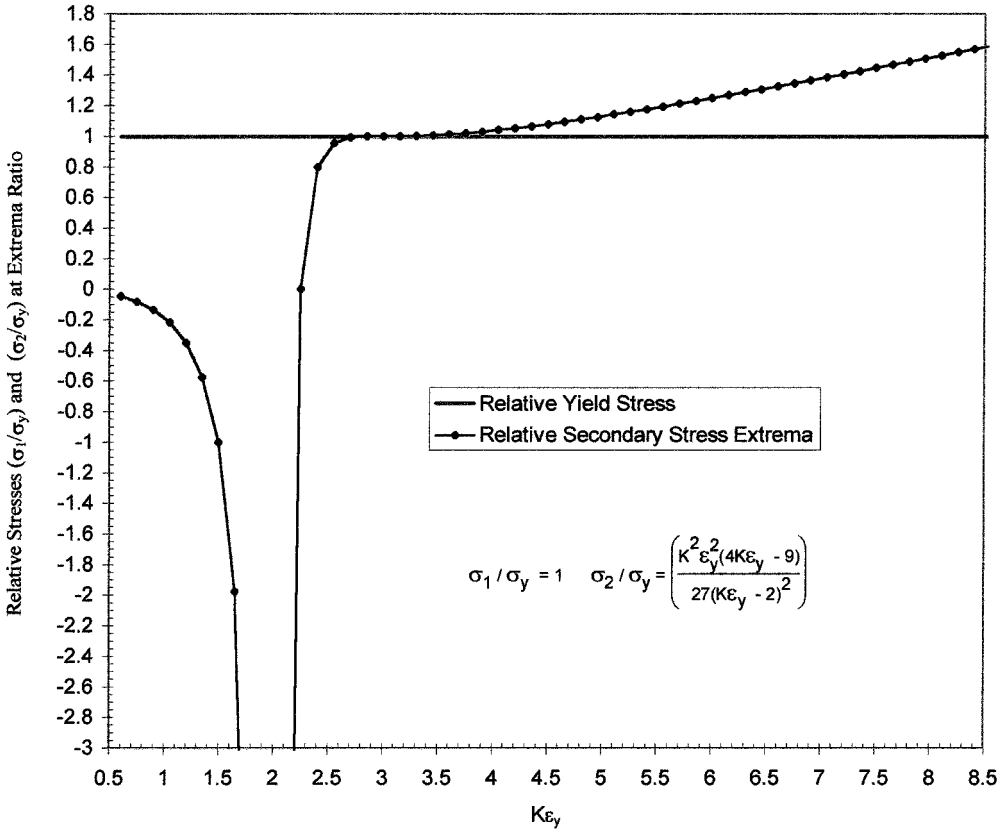

Figure 4 Relative yield stress (σ_1/σ_y) and relative secondary stress extrema (σ_2/σ_y) versus $K\varepsilon_y$.

Therefore, any strain, ε, in Figure 2 can be converted to a time, t, in Figure 5 if the associated strain rate, $\dot{\varepsilon}_i$, is known. Also note that the yield strain, ε_y, and the time to yield, t_y, are also related by a characteristic strain rate, $\dot{\varepsilon}_i$, as

$$t_y = \frac{\varepsilon_y}{\dot{\varepsilon}_i} \tag{15}$$

Substituting for ε_y from eq. (1) gives

$$t_y = \frac{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}{\dot{\varepsilon}_i} \tag{16}$$

Substituting eq. (16) into eq. (13) then gives

$$\sigma_y = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}\right)^n \tag{17}$$

Equation (17) can then be substituted into eq. (8) to give

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}\right)^n [K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{18}$$

Based on eqs. (17) and (18), it is apparent that any tensile stress, σ, associated with a specific strain value, ε, including the yield strength, σ_y, will increase with an increase in the strain rate, $\dot{\varepsilon}_i$. However, the strain to yield, ε_y, based on eq. (1), is only mildly sensitive to the strain rate and is allowed to either increase or decrease slightly with an increase in the strain rate, $\dot{\varepsilon}_i$. As indicated previously, these results are more consistent with the available data in the literature than is the approach suggested by Brown as discussed earlier.

However, it is interesting to address the case that exists at long times, t, or using eq. (18) at very low elongation rates, $\dot{\varepsilon}_i$. For this case, note that the yield stress, ε_y, approaches a limiting value, ε_∞:

$$\varepsilon_y = \varepsilon_\infty + \alpha\dot{\varepsilon}_i \rightarrow \varepsilon_\infty \quad \text{as} \quad \dot{\varepsilon}_i \rightarrow 0$$

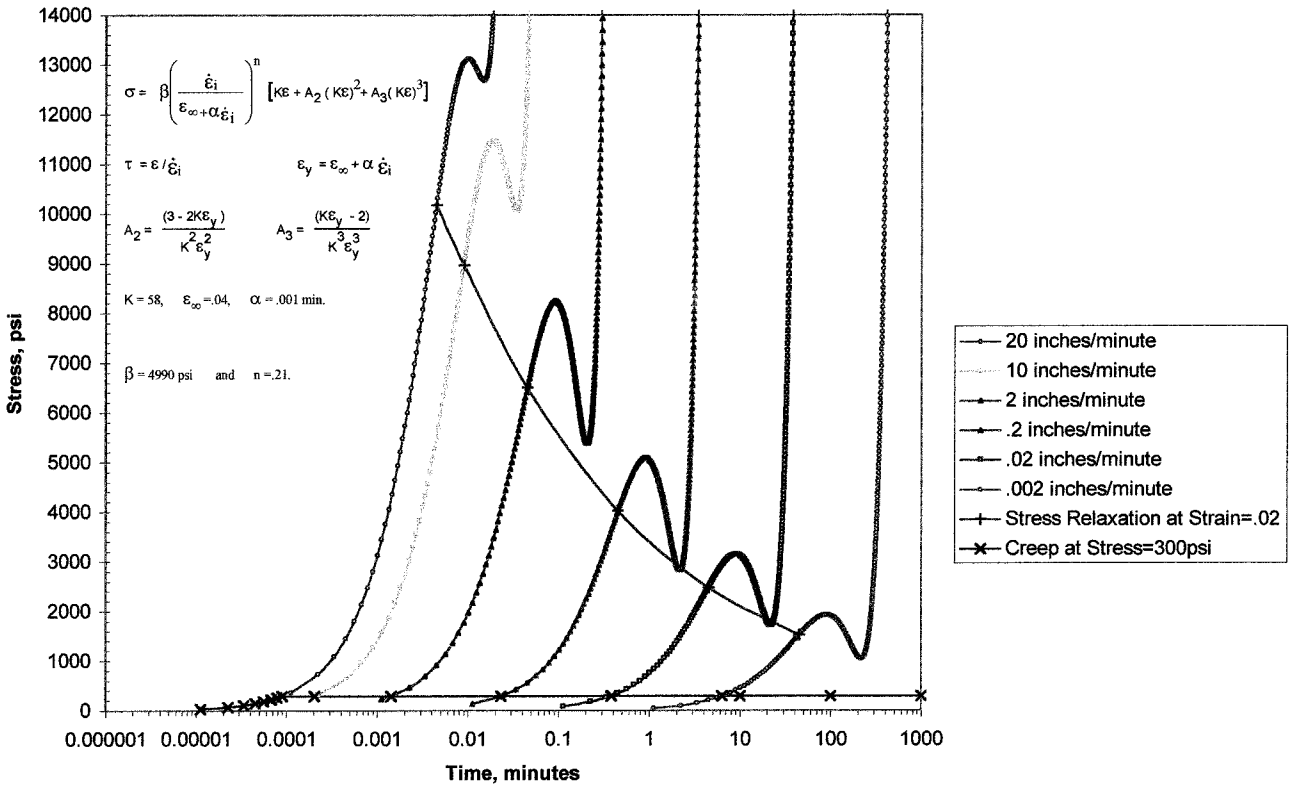

Figure 5 Calculated stress versus time at different strain rates using model from this study with indications of examples for creep and stress-relaxation contributions.

For this case, the constants A_2 and A_3 also approach the following values:

$$A_2 = \frac{(3 - 2K\varepsilon_\infty)}{K^2\varepsilon_\infty^2} \tag{19}$$

$$A_3 = \frac{(K\varepsilon_\infty - 2)}{K^3\varepsilon_\infty^3} \tag{20}$$

and eq. (18) reduces to

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty}\right)^n [K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{21}$$

Combining eqs. (14) and (21) gives

$$\sigma = \beta\left(\frac{\varepsilon}{\varepsilon_\infty}\right)^n \left(\frac{1}{t^n}\right)[K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{22}$$

Again, it should be noted that eqs. (21) and (22) apply only to the condition where the yield strain, ε_y, approaches its limiting value of ε_∞ as a result of the strain rate, $\dot{\varepsilon}_i$, approaching zero. However, as will be indicated shortly, both eqs. (21) and (22) can be extremely helpful when trying to address either creep or stress relaxation at very low strain rates, $\dot{\varepsilon}_i$, or a very long times, t.

In general, eqs. (1), (8)–(10), (17), and (18) can then be used to describe a complete series of uniaxial constant strain-rate curves for a given polymer formulation and/or processing condition as indicated in Figure 2. For reference, all the constant strain-rate curves in Figure 2 were generated using eq. (18) with the following typical parameters for an ABS-type polymeric material $K = 58$, $\varepsilon_\infty = 0.04$, $\alpha = 0.001$ min, $\beta = 4990$ psi, and $n = 0.21$. In addition, eqs. (1), (8)–(10), (17), and (18) can also be used to evaluate stress-relaxation and creep processes as will be indicated in the next two sections.

Stress-relaxation Analysis

Stress relaxation is defined as the time-dependent decay of stress in a viscoelastic material under a sustained and constant level of strain. As indicated in Figure 2, a stress-relaxation curve at

a specific strain can be described from a series of points at the same strain level collected from several different constant strain-rate curves. By connecting these same points as shown in Figure 5, it is easy to see that this series of points forms a stress-relaxation curve as a function of time. However, to initiate such a stress-relaxation process, the strain must first be increased to the desired level, ε_R, before the stress-relaxation process can begin. Equation (18) can then be used to generate this stress, σ, versus strain, ε, curve at a specific strain rate, $\dot{\varepsilon}_i$, until the desired level of strain, ε_R, is achieved from which stress relaxation will be initiated. Once the desired strain level, ε_R, has been reached, then eq. (18) needs to be used at a constant level of strain, ε_R, to predict the associated levels of stress for this stress-relaxation process as a function of time:

$$\sigma_R = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}\right)^n [K\varepsilon_R + A_2(K\varepsilon_R)^2 + A_3(K\varepsilon_R)^3] \tag{23}$$

After the desired strain level, ε_R, has been achieved, then successive stress values for the stress-relaxation process can be developed by identifying the appropriate stress on successive stress–strain curves that corresponds to the desired level of strain, ε_R, being evaluated. At this point, each constant strain-rate curve can be used to generate only one stress level, σ_R, at a given strain level, ε_R, on the stress-relaxation curve as indicated in Figure 2. Since each constant strain-rate curve can be described by eq. (23), then this equation can be used to calculate the stress, σ_R, at the desired strain, ε_R, and at the characteristic strain rate of $\dot{\varepsilon}_i$ being addressed. Note that for a stress-relaxation process both the yield strain, ε_y, and the yield strength, σ_y, are only functions of the strain rate, $\dot{\varepsilon}_i$, as indicated in eqs. (1) and (17). Also note that the relaxation time, t_R, accumulated for a specific relaxation strain, ε_R, at a specific strain rate, $\dot{\varepsilon}_i$, can be calculated directly from eq. (14). Therefore, at a constant relaxation strain level, ε_R, the stress-relaxation values for a series of stress levels, σ_R, and their associated relaxation time, t_R, can be calculated from a series of stress–strain curves. The locus of these points involving calculated values of stress, σ_R, and associated times, t_R, then constitutes the stress-relaxation curve. All these points are conveniently described by eq. (23), which then allows the stress-relaxation curve to be generated.

Also note that long relaxation times, t_R, are characterized by very small strain-rate values, $\dot{\varepsilon}_i$. In particular, at long relaxation times, t_R, and very small strain-rate values, $\dot{\varepsilon}_i$, the elongation to yield, ε_y, approaches the constant value, ε_∞, and the values of A_2 and A_3 in eq. (24) also approach values that do not change with an increase in time. This condition was previously shown to result in eq. (22), which can then be modified for stress-relaxation measurements as

$$\sigma_R = \beta\left(\frac{\varepsilon_R}{\varepsilon_\infty}\right)^n \left(\frac{1}{t_R^n}\right)[K\varepsilon_R + A_2(K\varepsilon_R)^2 + A_3(K\varepsilon_R)^3] \tag{24}$$

At long times, then, all the variables in eq. (24) are constants except the relaxation stress, σ_R, and the relaxation time, t_R. Equation (24) can then easily generate the remainder of the stress-relaxation curve at the desired relaxation times, t_R.

Using the formulation concepts discussed in this article, Figure 6 illustrates the initial phase of a stress-relaxation test beginning with the constant strain-rate component of this configuration followed by the more typical stress-relaxation process. Of particular interest is the observation that different phases of the curve in Figure 6 generate straight lines when plotted on a log–log scale as indicated in Figure 7. Note in Figure 7 that these stress-relaxation results can be described over a much larger time scale in a very convenient fashion. Of even more importance is the convenient and straightforward fashion in which it was possible to shift from a constant strain-rate configuration to a stress relaxation without changing formulation considerations. This approach should be very convenient for finite-element analysis evaluations.

Creep-curve Generation

Creep is defined as the time-dependent increase in strain of a viscous or viscoelastic material under sustained and constant stress. Again, as indicated in Figure 2, a creep curve can be developed from an identification of the strain at a series of points at the same stress level from a series of constant strain-rate curves. However, initially, the increase in stress to the level from which the creep curve can be initiated must be simulated. Typically, the simplest simulation approach can be achieved from a constant strain-rate process that can be used to achieve the desired level of

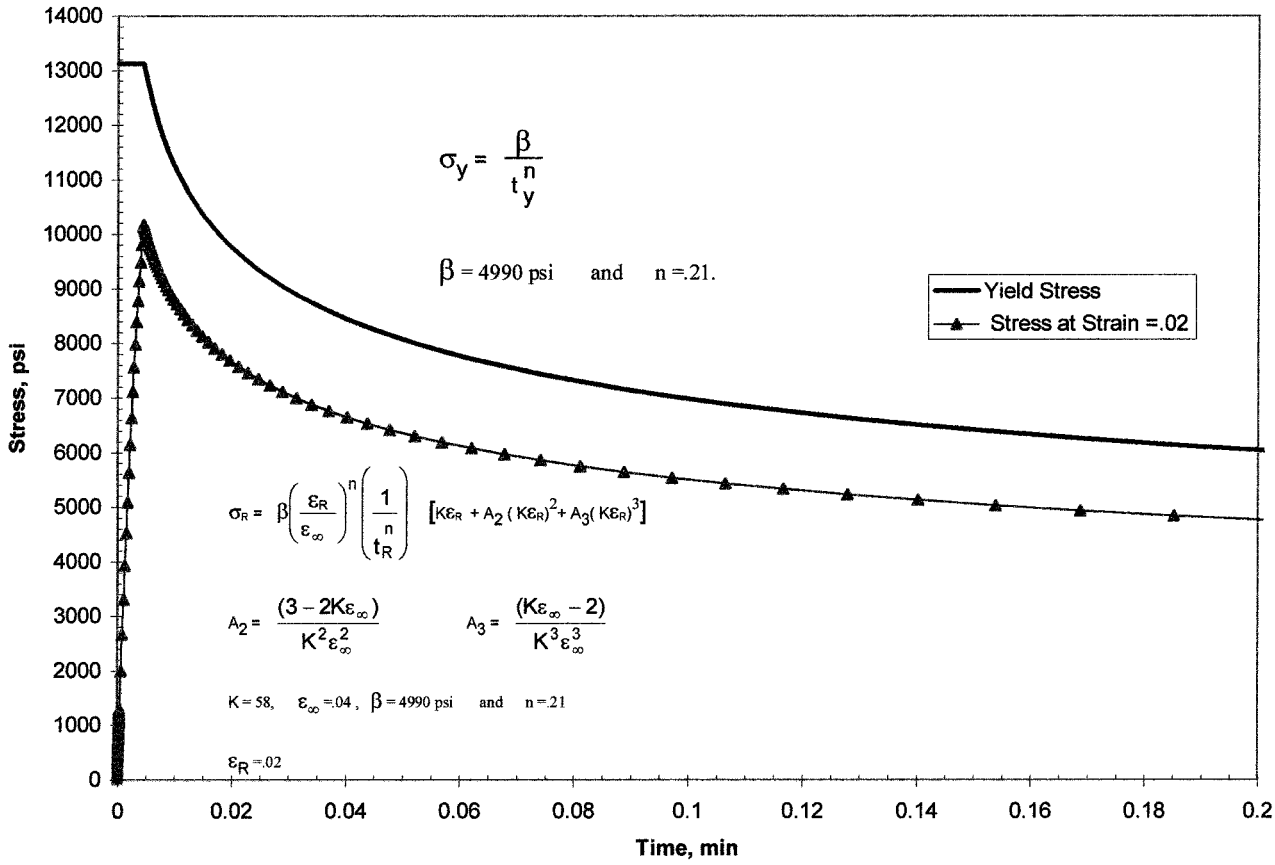

Figure 6 Constant strain rate used to achieve desired strain followed by short-term stress relaxation at 2% strain.

unchanging stress, σ_C, from which the creep process can begin. Similar to the case for stress relaxation, eq. (18) can then be used to generate this stress, σ, versus strain, ε, curve at a specific strain rate, $\dot{\varepsilon}_i$, until the desired level of stress, σ_C, is achieved from which a creep process can be initiated.

Once the desired stress level, σ_C, has been reached using a constant strain-rate approach, then eq. (18) must be solved for the strain to give a specific stress, σ_C, as the strain rate, $\dot{\varepsilon}_i$, is continued to be decreased to get to longer creep strains, ε_C, which can then be converted to creep times, t_C. Also, note that at a specific strain rate, $\dot{\varepsilon}_i$, the creep time, t_C, accumulated for a creep strain, ε_C, can be calculated directly from eq. (14). The locus of points involving calculated values of creep strain, ε_C, and the associated creep times, t_C, then constitutes the creep curve.

There are four potential ways then to calculate creep strain, ε_C, using eq. (18) as a function of the strain rate, $\dot{\varepsilon}_i$, to longer creep times, t_C. These four options include 1. Solve eq. (18) as a cubic equation to calculate the appropriate creep strain, ε_C, at decreasing levels of strain rate, $\dot{\varepsilon}_i$, but at the desired creep stress level, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (14). 2. Solve eq. (18) using a numerical method such as the Newton–Raphson method to calculate the appropriate creep strain, ε_C, at a decreasing levels of strain rate, $\dot{\varepsilon}_i$, but at the desired creep stress level, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (14). 3. Solve eq. (18) as a constant strain-rate evaluation for each strain rate, $\dot{\varepsilon}_i$, and then by solving eq. (18) by trial and error for the creep strain, ε_C, that yields the desired creep stress, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (14). 4. Assuming the controlling strain rates, $\dot{\varepsilon}_i$, are very small after the constant level of creep stress, σ_C, is achieved, and assuming the relative insensitivity of the values of A_2 and A_3 to

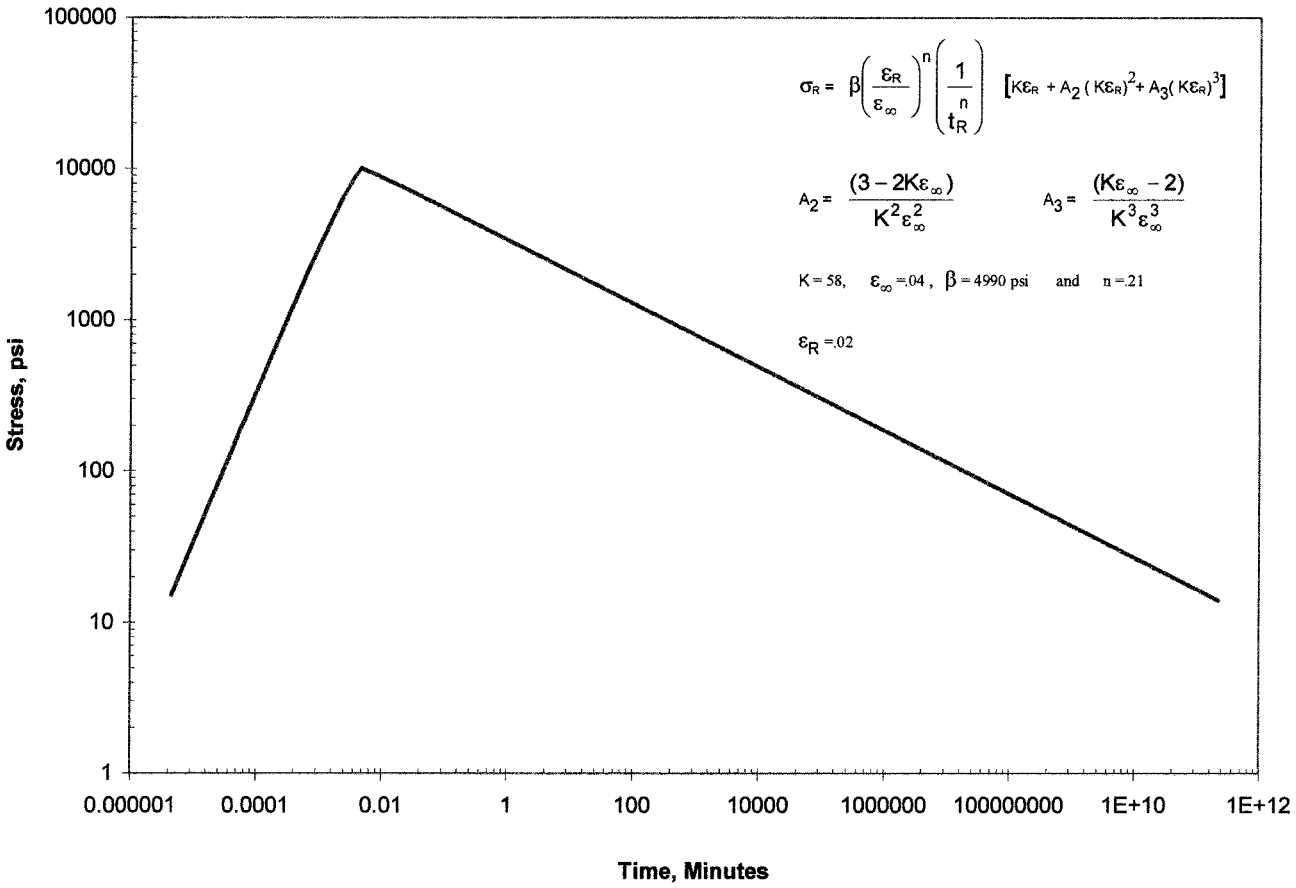

Figure 7 Calculated plot of long-term stress relaxation at 2% strain for this new model, showing the linear character of stress at both short and long times on a log–log scale.

the strain rate, then the creep curve can be calculated by close approximation directly using eq. (22) modified for creep to give

$$t_c = \left(\frac{\varepsilon c}{\varepsilon_\infty}\right)\left(\frac{\beta}{\sigma_c}\right)^{1/n}[K\varepsilon_c + A_2(K\varepsilon_c)^2 + A_3(K\varepsilon_c)^3]^{1/n} \tag{25}$$

By setting the creep stress, σ_C, to a constant value, then the creep time, t_C, can be calculated as a function of creep strain, ε_C, using eq. (25).

While method 3 appears to be very time-consuming, it can actually be relatively fast using a spreadsheet software such as MS Excel. This approach was also found to be particularly useful as the yield condition for creep or the inception of tertiary creep was approached and exceeded. If Option 3 is used, eq. (18) is first applied at a constant strain rate to increase the stress and associated strain until the desired stress level has been achieved. After the desired stress level has been reached, the successive strain values for the creep process can be developed by identifying the appropriate strain on successive stress–strain curves that corresponds to the desired level of stress being evaluated. At this point, each constant strain-rate curve can be used to generate only one strain level at a given stress level on the creep curve as indicated in Figure 2. Since each constant strain-rate curve can be described by eq. (18), then these equations can be used to calculate the strain, ε_C, at the desired stress, σ_C, and at the characteristic strain rate of $\dot{\varepsilon}_i$ being addressed. Again, note that for a creep process both the yield strain, ε_y, and the yield strength, σ_y, are functions of only the strain rate, $\dot{\varepsilon}_i$, as indicated in eqs. (1) and (17). Also note that the time, t_C, for that specific strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can be calculated directly from eq. (14). Therefore, at a constant

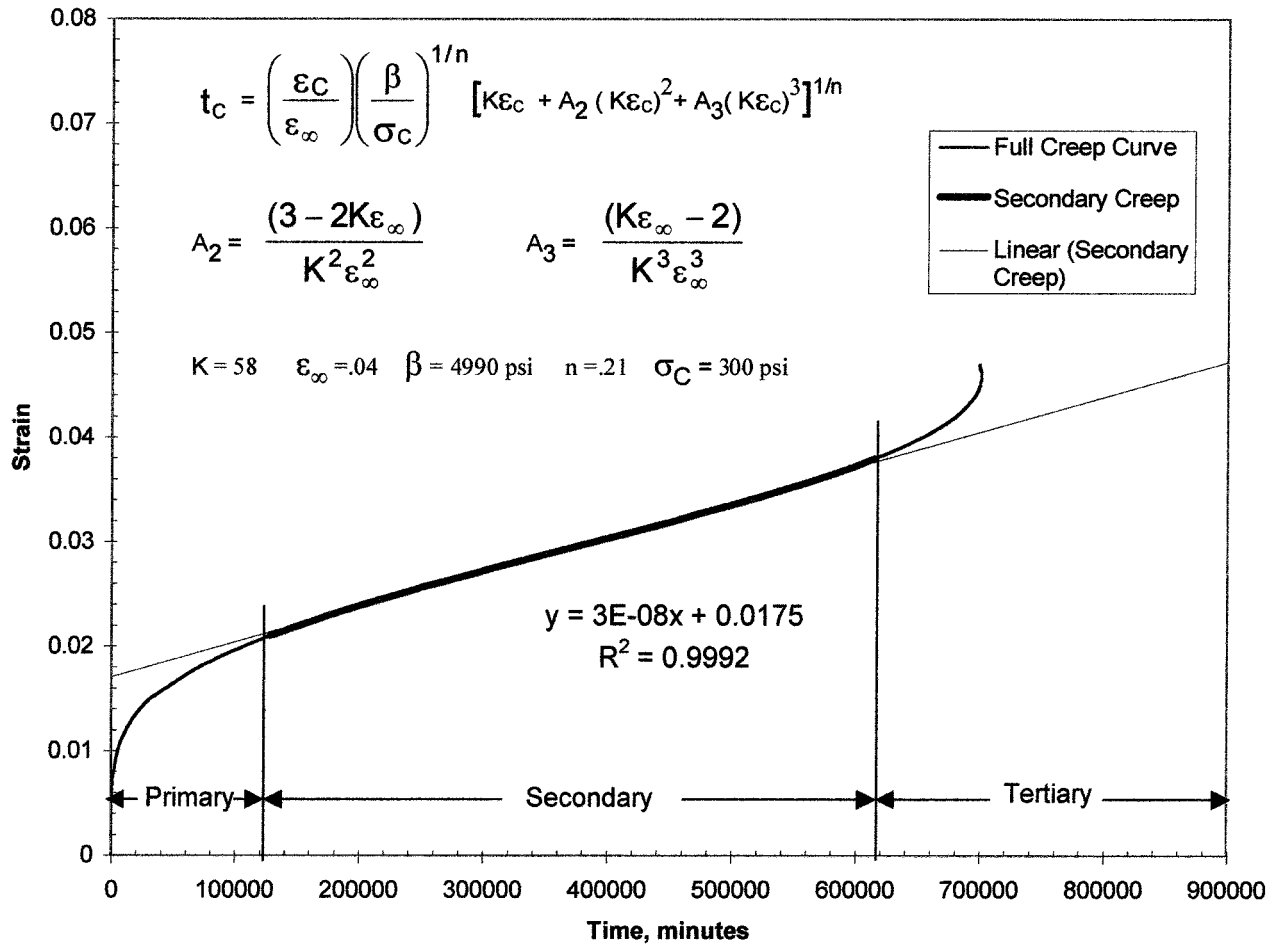

Figure 8 Calculated constant strain rate until desired stress (300 psi) was reached followed by creep strain versus time and showing all three phases of creep.

stress level, σ_C, the creep curve for a series of strain levels, ε_C, and their associated times, t_C, can be calculated from a series of constant strain-rate stress–strain curves. The locus of these calculated points then constitutes the creep curve as indicated in Figure 8.

If the simplifying assumptions of Option 4 are acceptable, the use of eq. (25) allows the simplest approach to generate all three phases of the creep curve including primary, secondary, and tertiary creep as indicated in both Figures 8 and 9. According to Thorkildsen,[18] primary creep includes all the initial changes in deformation prior to secondary creep. The first region of creep after primary creep that shows a linear increase in strain with time is called secondary creep. Tertiary creep is the final stage of creep and this stage of creep is often correlated with the yield point in making constant strain-rate measurements. Interestingly, it is easy to show with the model presented here that the yield point and the inception of tertiary creep are essentially manifestations of the same phenomena. In particular, if the strain to yield is assumed to be nearly a constant at very low crosshead speeds or at long creep times, then the strain to yield is essentially the same as is the strain at the inception of tertiary creep. Since the yield point for constant strain-rate data is considered in some circles to be at least one condition for failure in a polymer or polymer compound, then one potential failure condition for creep would be expected to begin at the inception of tertiary creep.

Using the formulation concepts to calculate creep as discussed in this article, the initial phase of a creep test begins with the constant strain-rate component followed by the more typical creep process as indicated in Figures 8 and 9. Of particular interest is the observation that the three different phases of the creep curve in Figure 8

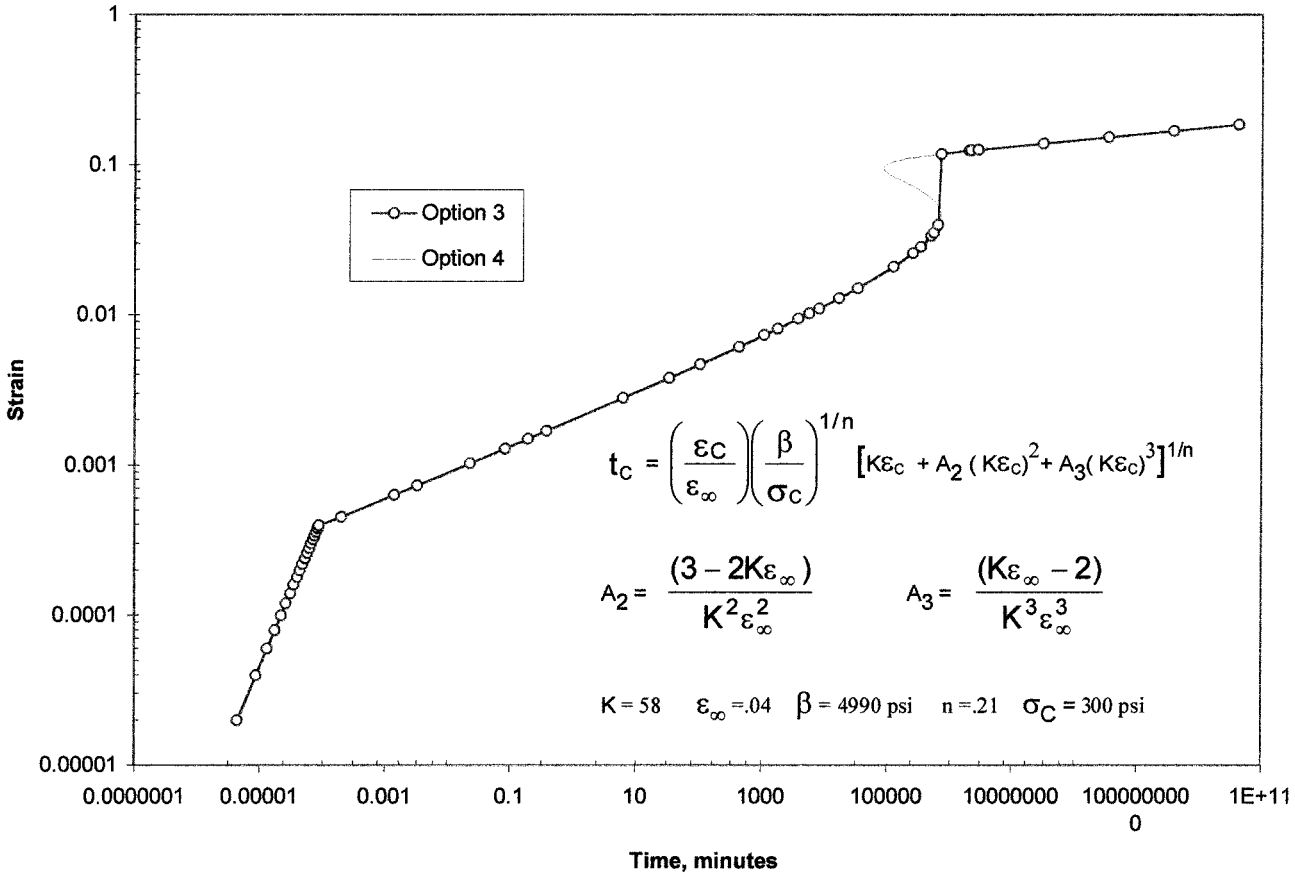

Figure 9 Calculated plot of long-term creep at 300 psi stress showing the linear character of strain at both short and long times on a log–log scale.

plot as straight lines when plotted on a log scale as indicated in Figure 9. Also, the creep results in Figure 9 have been described over a much larger time scale in a very convenient fashion. In addition, note in Figure 9 that both Options 3 and 4 give the same creep curve up until the inception of tertiary creep. At this point, there is a jump in the data using Option 3, but the results for Option 4 were essentially continuous over this same strain range. Beyond the yield strain or the inception of tertiary creep, both Options 3 and 4 gave approximately the same results but with some minor differences. While Option 4 yields much more continuous calculated results in this region, it has been found that both Options 3 and 4 predict a satisfactory value for the inception of tertiary creep.

CONCLUSIONS

This study has introduced a new unifying mathematical model to estimate creep, stress-relaxation, and/or constant strain evaluations as required for a given application for a viscoelastic and/or polymeric material. This new concept is especially significant since it adequately describes all the important characteristics of both creep and stress relaxation in the same model. In particular, all three phases of creep (i.e., primary, secondary, and tertiary) can be described adequately using this model. In addition, if the strain to yield at constant strain rates is assumed to be nearly a constant at very low crosshead speeds or at long creep times, then the strain to yield is essentially the same as the strain to the inception of tertiary creep. Thus, the model presented here clearly indicates that the yield point for constant strain-rate measurements and the inception of tertiary creep appear to be essentially manifestations of the same phenomena. Also, the initial buildup of stress followed by a drop off in stress as a function of time for stress relaxation is also adequately described by this model. This new model also offers a reasonably simple process in which to shift from a constant strain-rate config-

uration to a creep configuration or to a stress-relaxation configuration without changing formulation considerations. Most importantly, this model can be used to make a transition from one of these modes of stress configuration to another without any discontinuity in the stress or strain calculations. It is hoped that this analysis approach will open new doors for the design of plastic products for both short-term and long-term applications. It is expected that this approach should also be particularly convenient to introduce time influences for viscoelastic materials in finite-element analysis evaluations.

The author gratefully acknowledges Kenny Bertrand of Westlake Polymers for coordinating the supply and preparation of the polyethylene injection-molded tensile bars used in this study. The contribution of Westlake Polymers in supplying the polyethylene used in this study is also gratefully acknowledged. In addition, the author would also like to acknowledge Francisco (Paco) Canedo for evaluating the tensile measurements of these polyethylene samples as an undergraduate project at the University of Louisiana at Lafayette. Finally, the author would like to acknowledge Donesavanh Sabandith who pointed out the simplification that was possible for long-term creep or stress-relaxation predictions of the model presented in this article.

APPENDIX: THREE CONSTANT MECHANICAL PROPERTY MODEL MODIFICATION WHEN $K\varepsilon_Y > 3$

If $K\varepsilon_y > 3$, then, mathematically, it can be shown that the model presented in this article must be rewritten in the form

$$\frac{\sigma}{\sigma_y} = A_4[K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \quad \text{(A.1)}$$

where

$$K = \frac{A_1 E}{\sigma_y} \quad \text{(A.2)}$$

$$A_2 = \frac{(3 - 2K\varepsilon_c)}{K^2\varepsilon_c^2} \quad \text{(A.3)}$$

$$A_3 = \frac{(K\varepsilon_c - 2)}{K^3\varepsilon_c^3} \quad \text{(A.4)}$$

$$A_4 = \left(\frac{27(K\varepsilon_c - 2)^2}{K^2\varepsilon_c^2(4K\varepsilon_c - 9)}\right) \quad \text{(A.5)}$$

The two extrema are then identified as a maxima at

$$\sigma_1 = \sigma_y \text{ at } \varepsilon_1 = \varepsilon_y \quad \text{(A.6)}$$

and a minimum at

$$\sigma_2 = \sigma_y\left(\frac{27(K\varepsilon_c - 2)^2}{K^2\varepsilon_c^2(4K\varepsilon_c - 9)}\right) \text{ at } \varepsilon_2 = \varepsilon_c = \frac{3K\varepsilon_y + \sqrt{(3K\varepsilon_y)^2 - 24K\varepsilon_y}}{2K} \quad \text{(A.7)}$$

REFERENCES

1. Turner, S. Brit Plast 1964, Dec., 682–685.
2. Findley, W. N. Polym Eng Sci 1987, 27, 582–585.
3. Brinson, H. F.; DasGupta, A. Exp Mech 1975, Dec., 458–463.
4. Seitz, J. T. J Appl Polym Sci 1993, 49, 1331–1351.
5. Brown, N. Mater Sci Eng 1971, 8, 69–73.
6. Buchdahl, R. J Polym Sci A 1958, 28, 239–242.
7. Liu, Y.; Truss, R. W. J Polym Sci Part B Polym Phys 1994, 32, 2037–2047.
8. Mahfuz, H.; Zhu, Y. H.; Haque, A.; Abutalib, A.; Vaidya, U.; Jeelani, S.; Gama, B.; Gillespie, J.; Fink, B. Int J Impact Eng 2000, 24, 203–217.
9. Lee, S. W. R. Compos Sci Technol 1993, 49, 369–380.
10. Malpass, V. E. J Appl Polym Sci 1968, 12, 771–788.
11. Crochet, M. J. J Appl Mech 1966, 33, 326.
12. Brown, N. J Mater Sci 1983, 18, 2241.
13. Brown, N. In Failure of Plastics; Brostow, W.; Corneliussen, R. D., Eds.; Hanser: New York, 1986.
14. Robertson, R. E. General Electric Rep. No. 64-RL-(3580C), 1964.
15. Cigna, G.; Lomellini, P.; Merlotti, M. J Appl Polym Sci 1989, 37, 1527–1540.
16. English, L. ME 1988, May, 68–70.
17. Reinhart, F. W. Polym Eng Sci 1966, Oct., 28–295.
18. Thorkildsen, R.L. In Engineering Design for Plastics; Baer, E., Ed.; Robert E. Krieger: New York, 1975; Chapter 5.